

Ultra High Energy Decaying Fermions

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Abstract

We revisit the behaviour of fermions in the ultra high energy region with the recent approach of non commutative spacetime. It appears that at such high energies, particles could decay or show CPT violating behaviour in collisions. These considerations could be important in view of the fact that the LHC would be attaining an unprecedented 14TeV in 2013, hopefully.

1 Introduction

The LHC in Geneva is already operating at a total energy of $7TeV$ and hopefully after a pause in 2012, it will attain its full capacity of $14TeV$ in 2013. These are the highest energies achieved todate in any accelerator. It is against this backdrop that it is worthwhile to revisit very high energy collisions of Fermions (Cf. also [1]). We will in fact examine their behaviour at such energies.

2 The High Energy Equation

It is known that at very high energies, we encounter negative energy solutions. This is because the set of positive energy solutions of the Dirac or Klein-Gordon equations is not a complete set [2] and so cannot describe a particle localized in any sense. At usual energies we could apply the well known Foldy-Wouthuysen transformation to recover a description in terms of positive

energies alone or more precisely a description free of operators which mix negative energy and positive energy components of the wave function. This description also leads in the non-relativistic limit to the two component Pauli equation with the symbols E, p, γ and n having their usual meaning [3]. All this is well known.

In the case of very high energies it was shown several years ago by Cini and Toushek that we can modify the Foldy-Wouthuysen transformation and obtain a different description [4]. Let us examine this situation in greater detail [5]. The Cini-Toushek transformation can be written in the form

$$e^{\pm i s} = \frac{E + |p|}{2E} \pm \frac{\vec{\gamma} \cdot \vec{p}}{2E|p|} \cdot m \quad (1)$$

where E, p etc. have their usual meanings. Under (1), it is well known that the Dirac equation takes on the form of the massless neutrino equation:

$$H\psi = \frac{\vec{\alpha} \cdot \vec{p}}{|p|} E(p)\psi$$

In the above we use the following notation:

$$\alpha^k = \begin{pmatrix} 0 & \sigma^k \\ \sigma^k & 0 \end{pmatrix} \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (2)$$

$$\gamma^0 = \beta \quad (3)$$

$$\gamma^k = \beta \alpha^k \quad (k = 1, 2, 3) \quad (4)$$

where σ^k are the Pauli matrices and I is the 2×2 unit matrix.

We will also require the transformation of the γ_5 operator, which is, given by,

$$\gamma^5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3 = i \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (5)$$

Using (1), the transformed matrix is given by,

$$\Gamma_5 = e^{-i s} \gamma_5 e^{i s} = \left\{ \frac{E + p}{2E} + \frac{(\vec{\gamma} \cdot \vec{n})m}{2E} \right\} \gamma_5 \left\{ \frac{E + p}{2E} - \frac{(\vec{\gamma} \cdot \vec{n})m}{2E} \right\} \quad (6)$$

which finally reduced to

$$\Gamma_5 = \gamma_5 + \left(\frac{m}{E} \right) (\vec{\gamma} \cdot \vec{n}) \gamma_5 \quad (7)$$

In the above \vec{n} is the unit vector in the direction of the momentum vector. We can see from (7) that

$$\Gamma_5 = \gamma_5 \quad (8)$$

whenever m vanishes. This is of course the well known two component neutrino case where the wave function can be decomposed into the left handed and right handed neutrino wave functions. Let us use (7) to proceed along similar lines and write

$$\psi = \psi_1 + \psi_2 \quad (9)$$

where

$$\psi_1 = \frac{1}{2}(1 - \gamma_5)\psi \text{ and } \psi_2 = \frac{1}{2}(1 + \gamma_5)\psi \quad (10)$$

If (8) were to hold, as for the neutrinos, then (10) would be the decomposition in terms of the left handed and right handed wave functions. If the mass does not vanish, that is (8) does not hold then we will have from (10)

$$\psi_1 = (1 + \frac{m}{E})(1 + \gamma_5)\psi - \frac{m}{E}\psi \equiv (1 + \frac{m}{E})\psi_L - \frac{m}{E}\psi \quad (11)$$

with a similar equation for ψ_2 . Equations (9) and (11) show that if $\frac{m}{E}$ is much less than 1, that is when the energy is much greater than the rest energy, then we have a nearly two component neutrino like situation. We could for example interpret (9) and (10) as a decomposition into the left and right handed wave functions where the particle, as can be seen from (11) nearly exhibits handedness. Or more specifically as can be seen from (11) the wave function has a large part that displays handedness and a small part which is the usual type of wave function. More generally we can write (11) as

$$\psi = \psi_H + \omega\psi_\Delta \quad (12)$$

where ψ_H is the handed part and the second term is a small correction. It must be borne in mind that when the total energy is much greater than the rest energy (12) holds. One could hope to see the effects, hopefully in the LHC which as remarked has already reached the $7TeV$ mark and is expected to reach the $14TeV$ mark sometime in 2013.

3 Possible Consequences

Firstly, it must be observed that the above theory becomes relevant in view of the fact that the neutrino is now known to have a mass, though the

mass values are not yet certain, unlike the mass differences. This is because equations like (10), (11) and (12) can now be applied to neutrinos. This apart the above shows that Fermions in general behave like "heavy" neutrinos at very high energies. In any case as can be seen, these equations imply that apart from a $O(\frac{m}{E})$ correction, γ_5 gets multiplied effectively by a factor $(1 + O(\frac{m}{E}))$ (Cf.(11)). This means that in the usual Salam-Weinberg theory a typical interaction term gets multiplied by a factor $(1 + O(\frac{m}{E}))$ [6]

$$2^{\frac{1}{2}}G_w \left\{ \bar{\nu}_\mu \gamma^\lambda \frac{1}{2}(1 + \gamma_5)\nu_\mu \right\} \left\{ \bar{e} \gamma_\lambda \left[\frac{1}{2}(1 + \gamma_5)c_L + \frac{1}{2}(1 - \gamma_5)c_R \right] e \right\} (1 + O(\frac{m}{E})), \quad (13)$$

That is c_L and c_R are also multiplied by a similar small deviation from unity to become c'_L, c'_R . This in turn implies that the differential cross section now becomes, in terms of the fermion recoil energy E'

$$\begin{aligned} \frac{d\sigma}{dE'_0} &= [G_w^2/(2\pi m_e E_\nu^2)] [|c'_L|^2(p \cdot q)^2 + |c'_R|^2(p' \cdot q)^2 \\ &\quad + \frac{1}{2}(c'^*_R c'_L + c'^*_L c'_R)m_0^2 q \cdot q']. \end{aligned} \quad (14)$$

In any case the use of Γ_5 given by (7) instead of γ_5 would mean that a decay process would be asymmetrical in the angular distribution of the type $(1 + P \cos \Theta)$ where P is the average polarization.

The point is that fermions at such high energies would show handedness in accordance with (11) or (12). The possibility of CP violation in ultra high energy cosmic rays has been discussed by Collady and others [7]. In any case, these effects would have been present in the early universe.

Sudarshan et al [8] use a similar analysis to get positive and negative energy operators x_\pm for position and similar momentum operators, but interestingly they show that the x and y components do not commute. Sudarshan and co-workers introduced a sub or superscript D and E for the Dirac and extreme relativistic (that is Cini-Toushek type) representations. Then they deduced

$$\begin{aligned} [x_\pm, y_\pm] &= \left(\frac{ip_2}{2p^3} \gamma_5 \Lambda_\pm E \right)_{E \text{ repres.}} \\ &= \left(\pm \frac{\sigma_2}{2ip^2} \Lambda_\pm D \right)_{D \text{ repres.}} \end{aligned} \quad (15)$$

where Λ is a projection operator which is given by

$$\Lambda_{\pm} = \frac{1}{2}(1 \pm H/E)$$

in the considered representation. This matter was investigated earlier by Newton and Wigner too [9] from a slightly different angle. Some years ago the author revisited this aspect from yet another point of view [10] and showed that this noncommutativity which is exhibited by (15) is related to spin and extension. The noncommutative nature of spacetime has been a matter of renewed interest in recent years particularly in Quantum Gravity approaches. At very high energies, it has been argued that [11] there is a minimum fuzzy interval, symptomatic of a non commutative spacetime, so the usual energy momentum relation gets modified and becomes

$$E^2 = p^2 + m^2 + \alpha l^2 p^4 \quad (16)$$

the so called Snyder-Sidharth Hamiltonian [12, 13, 14]. Using (16) it is possible to deduce the ultrarelativistic Dirac equation [15]

$$(D + \beta l p^2 \gamma^5) \psi = 0 \quad (17)$$

$\beta = \sqrt{\alpha}$. In (17) D is the usual Dirac operator above while the extra term appears due to the new dispersion relation (16). We can see from (17) that the Hamiltonian now becomes non Hermitian and takes on an extra term (α being negative):

$$H = M - \imath N \quad (18)$$

where M is the usual Hamiltonian and N is now Hermitian (Cf.[16]), that is, M and N are real. This indicates a decay. With the modified Dirac equation (17) in place of the usual Dirac equation, we can now treat the two states considered above viz.,

$$\psi_L, \psi_R$$

as forming a two state system in this subspace of the Hilbert space of all states where the two components decay at different rates, in general as we will see below. The theory of such two state systems is well known [17]. In fact the two states would now be given by

$$\psi_{L,R}(t) = e^{\imath M t} \cdot e^{-N t} \psi_{L,R}(0) \quad (19)$$

where the left side refers to the state of time t and the right side wave function to the time $t = 0$ (Cf. also [18]). We can write the Hamiltonian (18) above for the two state as

$$H_{eff} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = M - \imath N = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \imath \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix}$$

where-by virtue of the pulled out \imath -both M and N are Hermitian. An additional constraint, namely $H_{11} = H_{22}$, comes from the CPT theorem. Let us continue with the two state analysis.

The evolution equation (in this sub space),

$$H|\psi\rangle = \imath \frac{d}{dt}|\psi\rangle$$

yields the usual solution

$$|\psi_{H,L}\rangle(t) = \exp[-\imath H_{H,L}]|\psi_{H,L}\rangle(0)$$

where $H_{H,L}$ denotes the eigenvalues of H , which are under the assumption of CPT symmetry given as is well known, by

$$H_{H,L} = H_{11} \pm \sqrt{H_{12}H_{21}}$$

and $|\psi_{H,L}\rangle$ are eigenstates of the form

$$|\psi_{H,L}\rangle = p|\psi^0\rangle \mp q|\bar{\psi}^0\rangle$$

with

$$\frac{q}{p} = -\frac{H_H - H_L}{2H_{12}}$$

Rewriting the time-dependent solution using $H_{H,L} = M_{H,L} - \imath N_{H,L}$ with real M and N , we get

$$|\psi_{H,L}\rangle(t) = \exp[-N_{H,L}] \exp[-\imath M_{H,L}](t) |\psi_{H,L}\rangle(0)$$

This represents two Fermions (one perhaps heavier with mass M_H , one lighter with mass M_L), decaying with (generally different) decay constants $N_{H,L}$. The mean mass $M = \frac{1}{2}(M_H + M_L)$ and $\Delta M = M_H - M_L$. It has been pointed out [19] that equations like (10), (11) or (12) applied to neutrinos which are massless suggests one (or more) neutrinos. This is brought out more clearly

in the above. Remarkably there seems to be very recent confirmation of such an extra or sterile neutrino [20].

In any case this analysis is true for Fermions in general and one would expect handedness and even decomposition at very high energies. One could look at it in the following way. The extra term in the new Hamiltonian (16), the modified Dirac equation (17) and the non Hermitian Hamiltonian (18) split the state, much like the introduction of a magnetic field leading to the Zeeman splitting.

Appendix

It is interesting that in the theory of Bosons too, we encounter a situation similar to that discussed above, with two states and a non Hermitian Hamiltonian. That is because in Quantum Mechanics we encounter negative energies, unlike in Classical Physics. In the case of the Dirac electron, this lead to the postulation of the Hole theory. Let us now start with the Klein-Gordon equation. As has been shown in detail by Feshbach and Villars [2], we can rewrite the K-G equation in the Schrodinger form, invoking a two component wave function,

$$\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad (20)$$

The equation is

$$\begin{aligned} i\hbar(\partial\phi/\partial t) &= (1/2m)(\hbar/i\nabla - eA/c)^2(\phi + \chi) \\ &\quad + (e\phi + mc^2)\phi, \\ i\hbar(\partial\chi/\partial t) &= -(1/2m)(\hbar/i\nabla - eA/c)^2(\phi + \chi) + (e\phi - mc^2)\phi \end{aligned} \quad (21)$$

It will be seen that the components ϕ and χ are coupled in (21). In fact we can analyse this matter further, considering free particle solutions for simplicity. We write,

$$\begin{aligned} \Psi &= \begin{pmatrix} \phi_0(p) \\ \chi_0(p) \end{pmatrix} e^{i/\hbar(p\cdot x - Et)} \\ \Psi &= \Psi_0(p) e^{i/\hbar(p\cdot x - Et)} \end{aligned} \quad (22)$$

Introducing (22) into (21) we obtain, two possible values for the energy E , viz.,

$$E = \pm E_p; \quad E_p = [(cp)^2 + (mc^2)^2]^{\frac{1}{2}} \quad (23)$$

The associated solutions are

$$\begin{aligned} \left. \begin{aligned} E = E_p \quad \phi_0^{(+)} &= \frac{E_p + mc^2}{2(mc^2 E_p)^{\frac{1}{2}}} \\ \psi_0^{(+)}(p) : \quad \chi_0^{(+)} &= \frac{mc^2 - E_p}{2(mc^2 E_p)^{\frac{1}{2}}} \end{aligned} \right\} \phi_0^2 - \chi_0^2 = 1, \\ \left. \begin{aligned} E = -E_p \quad \phi_0^{(-)} &= \frac{mc^2 - E_p}{2(mc^2 E_p)^{\frac{1}{2}}} \\ \psi_0^{(-)}(p) : \quad \chi_0^{(-)} &= \frac{E_p + mc^2}{2(mc^2 E_p)^{\frac{1}{2}}} \end{aligned} \right\} \phi_0^2 - \chi_0^2 = -1 \end{aligned} \quad (24)$$

It can be seen from this that even if we take the positive sign for the energy in (23), the ϕ and χ components get interchanged with a sign change for the energy. Furthermore we can easily show from this that in the non relativistic limit, the χ component is suppressed by order $(p/mc)^2$ compared to the ϕ component exactly as in the case of the Dirac equation [3]. Let us investigate this circumstance further. In (21) if we take

$$\frac{1}{2m}(-i\nabla - eA)^2\chi = 0 \quad (25)$$

then we have

$$i\dot{\phi} = \frac{1}{2m}(-i\nabla - eA)^2\phi + (eA_0 + mc^2)\phi \quad (26)$$

and also

$$\begin{aligned} i\dot{\chi} &= -\frac{1}{2m}(-i\vec{\nabla} - cA)^2\phi + (eA_0 - mc^2)\chi \\ &= -i\dot{\phi} + (eA_0 + mc^2)\phi + (eA_0 - mc^2)\chi \end{aligned} \quad (27)$$

It can be seen that (25) and (26) are Schrodinger equations and so solvable. However (27) couples ϕ and χ . In fact we have

$$\dot{\phi} + \dot{\chi} = (eA_0 + mc^2)(\phi + \chi) - 2mc^2\chi \quad (28)$$

In the case if

$$mc^2 \gg eA_0 \quad (\text{or } A_0 = 0) \quad (29)$$

we can easily verify that

$$\phi = e^{ipx - Et} \text{ and } \chi = e^{ipx + Et} \quad (30)$$

is a solution.

That is ϕ and χ belong to opposite values of $E(m \neq 0)$. The above shows that K-G equation mixes the positive and negative energy solutions. Solutions with predominantly positive energies represent particles while those where negative energies predominate represent anti particles. If on the other hand $|mc^2| \ll 1$, then (24) shows that χ and ϕ are of same energy that is $m_0c^2 \approx 0$ that is $m_0 = 0$. This shows that if ϕ and χ both have the same sign for E , that is there is no mixing of positive and negative energy, then the rest mass m_0 vanishes. Further we are now in a position to argue that solutions with a single sign of the energy have no rest mass. A non vanishing rest mass requires the mixing of both signs of energy. Indeed it is a well known fact

that for solutions which are localized about a point x_0 in the δ function sense, both signs of the energy solutions are required to be superposed [5]. This is because only positive energy solutions or only negative energy solutions do not form a complete set. Interestingly the same is true for localization about a time instant t_0 .

Further, we observe that

$$t \rightarrow -t \Rightarrow E \rightarrow -E, \quad \phi \leftrightarrow \chi \quad (31)$$

Let us write (27) as ‘(with $\hbar = 1 = c$)

$$H\phi = H_{11}\phi + H_{12}\chi \quad (32)$$

and similarly we have

$$H\chi = H_{21}\phi + H_{22}\chi \quad (33)$$

We now observe that in Quantum Field Theory, a sub space of the full Hilbert can exhibit the complex or non Hermitian Hamiltonian of the type encountered above.

Writing $H = M - \imath N$ as before we have

$$\begin{aligned} M_{11} &= -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2m}\frac{e^2 A^2}{c^2} + (e\phi + mc^2) \\ M_{21} &= +\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2 A^2}{c^2} + (e\phi - mc^2) \\ N_{11} &= \frac{1}{m}\frac{eA}{c}\hbar\nabla = N_{12} \\ N_{21} &= -N_{11} = N_{22} \end{aligned} \quad (34)$$

We can now treat $|\phi, \chi\rangle$ as a two state system and further it follows from the above that

$$\begin{aligned} |\phi, \chi\rangle(t) &= \exp(-N_{12}t)\exp. \\ &\exp(-\imath M_{12}t) |\phi, \chi\rangle(0) \end{aligned} \quad (35)$$

Equation (35) shows that the states $|\phi\rangle$ and $|\chi\rangle$ decay, but decay at different rates.

Treating $|\phi\rangle$ and $|\chi\rangle$ as particle and anti particle, we have exactly this situation in B and K^0 decay. The point here is that as in the case of the B or K^0 mesons, the decay rates of the particles and antiparticles would be different, thus leading to a CPT violation. The above considerations provide an explanation. A full discussion will be given later. This work was partly supported by a grant from the Santilli Foundation

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